Physics 287

More Intermittency

A.)

→ Why the Fractology and what do we get from β-Model?

- $\boxed{\text{higher moments are a more severe probe of small scale structure of turbulence than energy is!}}$

Recall K41 ⇒ $\delta v(\ell) \sim \epsilon^{1/3} \ell^{1/3}$

$$\therefore \quad \langle \delta v(\ell)^p \rangle \sim \epsilon^{p/3} \ell^{p/3}$$

so normalizing:

$$\langle \delta v(\ell)^p \rangle \Big/ \langle \delta v(\ell)^2 \rangle^{p/2} \sim 1$$

normalized moments all independent of scale! → Testable Prediction

So, what of higher moments, i.e. $p > 2$?

Special interest in:

$\rightarrow$ $p = 3$ - skewness S (measure of symmetry)

why? turbulence $\leftrightarrow$ statistical approach/picture

naively: Gaussian distribution (i.e. random)

so $S \to 0$.

but:

$$\partial_t E \sim \partial_t V^2 \sim V^3 \quad ; \quad \underline{\underline{so}}$$

net energy transfer in cascade, and

$$\langle v^3 \rangle \neq 0.$$

Similarly, $p = 4$ - kurtosis K

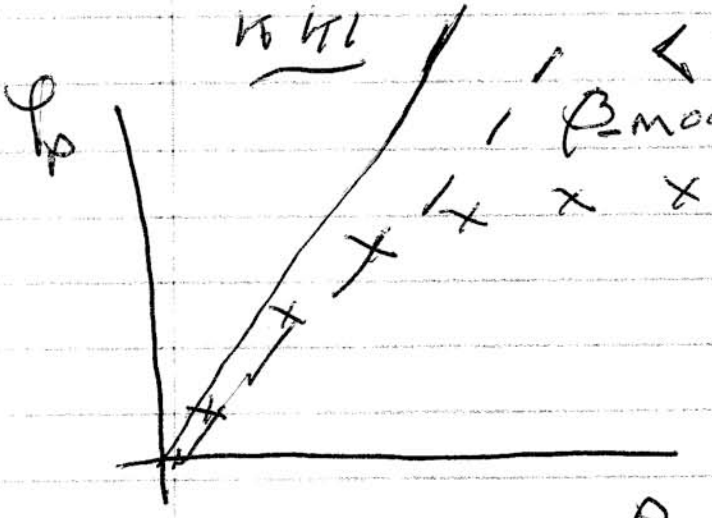
$$K = \langle \delta V^4 \rangle / \langle \delta V^2 \rangle^2 \rightarrow 3 \text{ for Gaussian}$$

process. $\Rightarrow$ measure of importance/weight of <u>tails</u> of distribution

$\Rightarrow$

$K \gg 1$ is indicative of strong correlations and non-Gaussian behavior, and fat tails.

→ The Data (mostly in time, i.e. $\langle V(t)^p \rangle$ etc.)



⇒ reality departs K41!

→ What does β-model predict?

volume factor

$$\langle |\delta v(\ell_n)|^p \rangle \sim \beta_n \left[\delta v(\ell_n)\right]^p$$

↳ set by cascade scaling

$$\sim \epsilon^{p/3}\, \ell_n^{p/3} \left(\ell_n / \ell_0\right)^{\zeta_p}$$

wt factor

exponent of intermittency correction

$$\zeta_p = \frac{1}{3}(3-D)(3-p)$$

δv^p.

so, normalization $\Rightarrow$

$$a_p(\ell_n) \sim \langle \delta v(\ell_n)^p \rangle / \langle \delta v(\ell_n)^2 \rangle^{p/2}$$

plugging in $\Rightarrow$

$$\boxed{\begin{aligned} a_p(\ell_n) &\sim (\ell_n/\ell_0)^{\varepsilon_p} \\ \varepsilon_p &= 1/2\,(3-D)(2-p) \end{aligned}}$$

n.b.
$p > 2$
$\varepsilon_p < 0$

In particular:

$$S \sim \langle \psi^3 \rangle / \langle \psi^2 \rangle^{3/2}$$

$\psi \sim \nabla v$

$$\sim Re^{3(3-D)/2(1+D)}$$

taking $\ell_n \sim \ell_d$
as effect maximal.

$$K \sim S^2$$

Note:

- departure from K41 strongest at smaller scales
  $\Rightarrow$ 'focus' of cascade strongest

- β model $\Rightarrow$ stages in cascade have "memory" of initial scale l_0

  $\Rightarrow$ l_0 explicit, beyond ϵ.

- $D = 2.8$ is reasonable data fit

  $\Rightarrow$ dissipative structure is highly convoluted sheets.

- τ_p departs β-model as $p \uparrow$

  $\Rightarrow$ Multi-fractal model ; i.e.

  β-model $\rightarrow$ single dissipative structure of dimension D.

  multi-fractal $\rightarrow$ multiple dissipative structures, different

$\Rightarrow$ connection to Navier-Stokes equation and dynamics is increasingly obscure ----

– a natural question:

→ have argued that intermittency

⇔ departure from simple, self-similarity scalings

⇒ manifested as $\ln l/l_0$ "memory" in structure function

→ have also stressed analogy between

self-similarity in space (Blast wave)

vs self-similarity in scale (K41).

∴

→ so, what is analogue of intermittency for space-time similarity, i.e.

K41 ⟷ β-model

as

Spatio-temporal self-similarity ⟷ ?

$\Rightarrow$ Memory of initial condition!

i.e. $F \rightarrow F(\underbrace{r/\rho(t)}_{\text{self-sim. variable}}, r_0)$

Note for Sedov-Taylor effectively ignored initial radius of blast!

$\Rightarrow$ { See Barenblatt, "Scaling"
Chapt. 3

Now, one can go further and calculate:

$\langle \tilde{\epsilon}^2 \rangle \rightarrow$ mean square fluctuating dissipation

but $\epsilon \sim \nu \langle (\nabla \tilde{v})^2 \rangle$

so $\langle \tilde{\epsilon}^2 \rangle \sim \nu^2 \langle (\nabla \tilde{v})^2 (\nabla \tilde{v})^2 \rangle \sim \nu^2 \beta_{l_d} \dfrac{\tilde{v}(l_d)^4}{l_d^4}$

if normalize:

$\langle \tilde{\epsilon}^2 \rangle / \langle \epsilon \rangle^2 \sim \cancel{\nu^2} K$

$\sim \cancel{\nu} Re^{3(3-D)/(1+D)}$ } Kurtosis!

Can also address:

$$\boxed{\langle \epsilon(\underline{r})\, \epsilon(\underline{r}+\underline{\ell})\rangle \rightarrow \text{dissipation correlation}}$$

Now,

$$\langle \epsilon(\underline{r})\, \epsilon(\underline{r}+\underline{\ell})\rangle \sim \langle \epsilon\rangle^2 \, \text{Prob}(\underline{r}, \underline{r}+\underline{\ell} \text{ belong to } n\text{-eddy})$$

$$\langle \epsilon \rangle \sim v_n^3 / \ell_n$$

$$\sim v(\ell_n)^3 / \ell_n$$

⇒ allowing for:

- packing
- if correlated by ℓ_n then correlated by all larger eddies

⇒

$$\langle \epsilon(r)\, \epsilon(r+\ell)\rangle \sim \sum_{n=0}^{\ell_n \lesssim \ell} \left(\frac{v_n^3(\ell_n)}{\ell_n} \right)^2 \beta_n$$

$$\sim \bar{\epsilon}^2 (\ell/\ell_0)^{-(3-D)}$$

In particular,

$$\boxed{\langle \epsilon(r)\, \epsilon(r+l_d) \rangle \sim \bar{\epsilon}^2 (l_d/l_0)^{(D-3)}}$$

$\Rightarrow$ strong correlation in dissipation at dispn. scale.